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REFLECTION OF ELECTROMAGNETIC WAVES FROM SUBIONOSPHERIC IONIZED LAYERS PRODUCED BY INTENSE ELECTROMAGNETIC BEAMS

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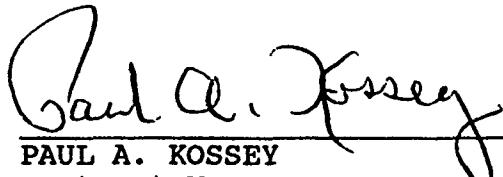
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| <p>Ionospheric layers produced by intense electromagnetic beams are characterized by an electron density profile that is nearly exponential in the lower boundary and an electron collision frequency that is nearly uniform. These features allow such layers to be approximated by Epstein profiles, which afford analytical exact full-wave reflection coefficients subject to only two restrictions: (1) the incident wave is horizontally polarized, and (2) the layer is stratified. Simple criteria for good reflectivity are derived from the reflection-coefficient formulas. For overdense plasma layers having plasma frequencies substantially above the wave frequency, the reflection loss is approximately $-5.8 \times 10^{-8} \text{ Lcos} \theta \text{ dB}$ where L is the electron collision frequency at the layer altitude, $1/\text{L}$ is the height-gradient of the electron density in the layer boundary, and θ is the angle of incidence. Because calculations neglect roughness, which could be caused by uneven illumination by the ionizing beam, the conditions for good reflection presented in this report should be regarded as necessary--but not sufficient. <i>Keywords:</i></p> | | |
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SUMMARY

Ionospheric layers produced by intense electromagnetic beams are characterized by an electron density profile that is nearly exponential in the lower boundary and an electron collision frequency that is nearly uniform. These features allow such layers to be approximated by Epstein profiles, which afford analytical exact full-wave reflection coefficients subject to only two restrictions: (1) the incident wave is horizontally polarized, and (2) the layer is stratified. Simple criteria for good reflectivity are derived from the reflection-coefficient formulas. For overdense plasma layers having plasma frequencies substantially above the wave frequency, the reflection loss is approximately $-5.8 \times 10^{-8} \nu L \cos \theta$ dB where ν is the electron collision frequency at the layer altitude, $1/L$ is the height-gradient of the electron density in the layer boundary, and θ is the angle of incidence. For layers created with intense 500 MHz beams, the scale length L is about 70 m, and reflection losses greater than 20 dB will be suffered unless the layer is high enough that the collision frequency ν does not exceed about 5×10^6 Hz. If the reflection occurs during the time between ionizing pulses, the layer must be at or above 70 to 75 km. If, however, reflection must occur when the ionizing beam is on, the layer must be above 110 km to avoid prohibitive reflection loss. Because calculations neglect roughness, which could be caused by uneven illumination by the ionizing beam, the above conditions for good reflection should be regarded as necessary--but not sufficient.

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SECTION I INTRODUCTION

The possibility of using intense microwave beams to create an ionized reflecting layer below the normal ionosphere was first suggested by Gurevich [1979, 1980]. Although various configurations can be envisioned, all such schemes would use large ground-based transmitters to radiate electric fields that exceed the breakdown field of air at some specified altitude, probably below about 100 km. The ensuing avalanche of electrons would create an ionized layer dense enough to reflect radio waves back to earth that would ordinarily pass through the ambient ionosphere and escape into space. A recently published Soviet book examines this subject in detail [Borisov, Gurevich and Milikh, 1989].

In assessing the reflectivity of such artificially produced ionized layers, the Soviets tacitly assume that reflection will occur provided the electron density in the layer is so high that the plasma frequency--adjusted for obliquity--exceeds the frequency of the wave to be reflected. That assumption--reflection at the classical turning point--is valid for all cases studied by the Soviets, who considered only wave frequencies above several hundred megahertz--always much higher than the electron collision frequency in the artificial layer.

More recently, American scientists have become interested in using an artificial layer to reflect waves having frequencies between 15 to 50 MHz. These signals have frequencies comparable to the collision frequency in the 50 to 90 km altitude range where the layer would most likely be created. Because of that high ratio of collision frequency to wave frequency, it is no longer true that an artificial ionized layer will reflect the signal simply because the plasma frequency exceeds the wave frequency. In fact, the reflectivity depends in a complicated way on several factors, including the product of the wavelength and the height-gradient of the refractive index in the layer boundary. This report examines this reflectivity in detail.

Our approach makes use of the fact that the layer's lower boundary, from which the wave is reflected, is characterized by an

electron-density profile that is nearly exponential and a collision frequency that is nearly uniform. These features allow us to approximate the layer closely with well-known Epstein profiles [Epstein, 1930], which afford exact full-wave expressions for the reflectivity, subject to only two restrictions: (1) the wave is horizontally polarized, and (2) the layer is stratified. The first condition poses no problem because horizontal is the preferred polarization for certain over-the-horizon (OTH) radar applications. The second condition, stratification, is probably violated because it is difficult to create a smooth layer. However, layer irregularities would most likely degrade the reflectivity in the specular direction, so our results comprise a probable upper bound on layer reflectivity.

Section II discusses modeling artificial layers with Epstein profiles; Sec. III reviews reflection formulas; Sec. IV states criteria that must be met to achieve good reflectivity; Sec. V presents graphs of reflectivity versus frequency and layer altitude for layers calculated under AFGL's Artificial Ionospheric Mirror (AIM) program; and Sec. VI states the conclusions.

SECTION II
MODELING ARTIFICIAL IONOSPHERIC LAYERS WITH EPSTEIN PROFILES

The equation for a horizontally polarized plane wave incident at angle θ onto an isotropic ionosphere of refractive index $n(z)$ is well known:

$$\left[\frac{d^2}{dz^2} + n^2(z) - \sin^2 \theta \right] E(z, \omega) = 0 \quad . \quad (1)$$

where E is the electric field, $\omega = 2\pi f$, and f is the wave frequency. Epstein [1930] showed that if

$$n^2 = 1 + \left\{ \frac{q_\infty^2 - \cos^2 \theta}{\exp(z/\sigma + b)} + \frac{A}{[\exp(z/\sigma + b) + 1]^2} \right\} \exp \left(\frac{z}{\sigma} + b \right) \quad (2)$$

where $q^2 = n^2 - \sin^2 \theta$ is the root of the Booker quartic, then Eq. (1) becomes the hypergeometric equation and can be solved analytically for reflection and transmission coefficients. A remarkable number of realistic ionosphere profiles can be constructed and solved exactly by judiciously selecting the parameters σ , b , and A in Eq. (2). We consider two such profiles here--the "thin layer" model and the "soft boundary" model.

THIN LAYER WITH EXPONENTIAL EDGES AND A CONSTANT COLLISION FREQUENCY

Make the following substitutions in Eq. (2):

$$A = \frac{4X_0}{1 - iZ_0} \quad (3a)$$

$$\sigma = L \quad (3b)$$

$$b = - \frac{z_o}{L} \quad (3c)$$

$$q_\infty = \cos \theta \quad (3d)$$

where $X_o = \omega_{po}^2/\omega^2$ and $Z_o = \nu/\omega$ are well-known magnetoionic variables involving the angular plasma frequency ω_p and collision frequency ν . Both ω_{po} and Z_o are constant. Those substitutions cause the refractive index to become

$$n^2 = 1 - \frac{4X_o}{1 - iz_o} \left[\frac{\exp [(z - z_o)/L]}{(\exp [(z - z_o)/L] + 1)^2} \right]. \quad (4)$$

Note that $\omega_p^2 = 3.2 \times 10^9 N$, so X_o is proportional to the electron density N_o at the reference height z_o .

The height-dependence of the electron density $N(z)$ is contained in the exponential terms within the large brackets:

$$N(z) = 4N_o \left[\frac{\exp [(z - z_o)/L]}{(\exp [(z - z_o)/L] + 1)^2} \right] \quad (5)$$

The following behavior occurs at low and high altitudes:

$$N(z) \approx 4N_o \exp [(z - z_o)/L] \quad \text{if} \quad (z - z_o)/L \ll -1 \quad (5a)$$

$$N(z) \approx 4N_o \exp [-(z - z_o)/L] \quad \text{if} \quad (z - z_o)/L \gg 1. \quad (5b)$$

We see, therefore, that the electron density corresponds to a symmetric layer whose width is on the order $2L$, and whose upper and lower boundaries decay exponentially with scale length L . This Epstein layer is sometimes called the "SECH²" profile [Budden, 1961; Ginzburg, 1970].

Figures 1 and 2 show Epstein thin layer profiles for layers centered at $z_0 = 61.4$ km and 72.7 km. The boundary scale-heights, respectively, are $L = 68.6$ m and 73.4 m. The reason for selecting these parameters is given below.

UNIFORM LAYER WITH GRADUAL LOWER BOUNDARY

To get another representation of an ionized layer, we make the following substitutions into Eq. (2):

$$A = 0 \quad (6a)$$

$$\sigma = L \quad (6b)$$

$$b = - \frac{z_0}{L} \quad (6c)$$

$$q_\infty = \sqrt{\cos^2 \theta - \frac{x_\infty}{1 - iz_0}} \quad (6d)$$

in which case the refractive index becomes

$$n^2 = 1 + \left(q_\infty^2 - \cos^2 \theta \right) \left\{ \frac{\exp \left[(z - z_0)/L \right]}{\exp \left[(z - z_0)/L \right] + 1} \right\} \quad (7a)$$

$$\therefore 1 - \frac{x_\infty}{1 - iz_0} \left\{ \frac{\exp \left[(z - z_0)/L \right]}{\exp \left[(z - z_0)/L \right] + 1} \right\} \quad . \quad (7b)$$

The electron density in this layer behaves as

$$N(z) = N_\infty \left\{ \frac{\exp \left[(z - z_0)/L \right]}{\exp \left[(z - z_0)/L \right] + 1} \right\} \quad (8)$$

$$\approx N_\infty \exp \left[(z - z_0)/L \right] \quad \text{if} \quad (z - z_0)/L \ll -1 \quad (8a)$$

$$\approx N_\infty \quad \text{if} \quad (z - z_0)/L \gg 1 \quad . \quad (8b)$$

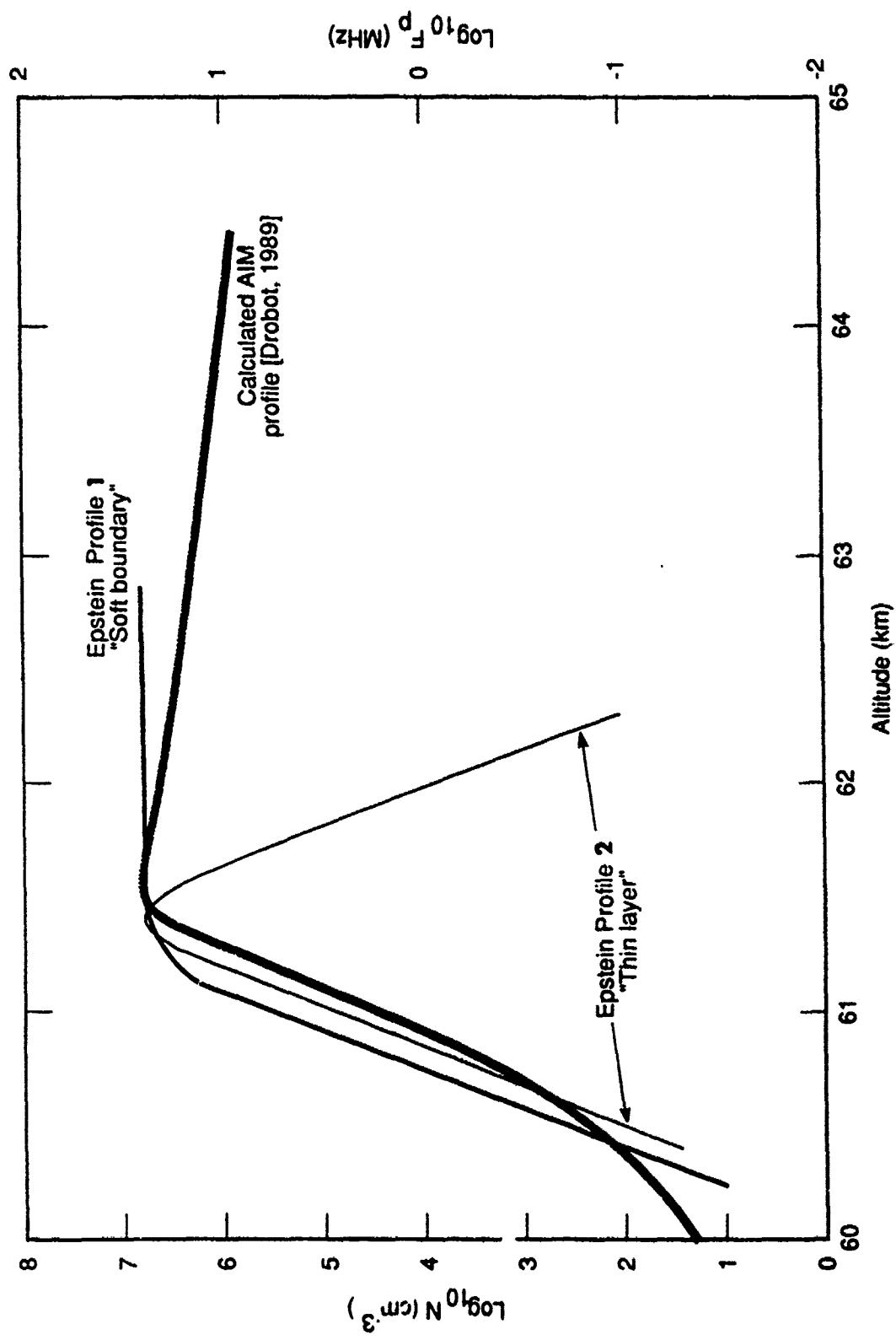


Figure 1. Comparison of Epstein "Thin Layer" and "Soft Boundary" Profiles with Calculated AIM Profile Peaking at an Altitude of 61.4 km; Boundary Scale $L = 72.3$ m.

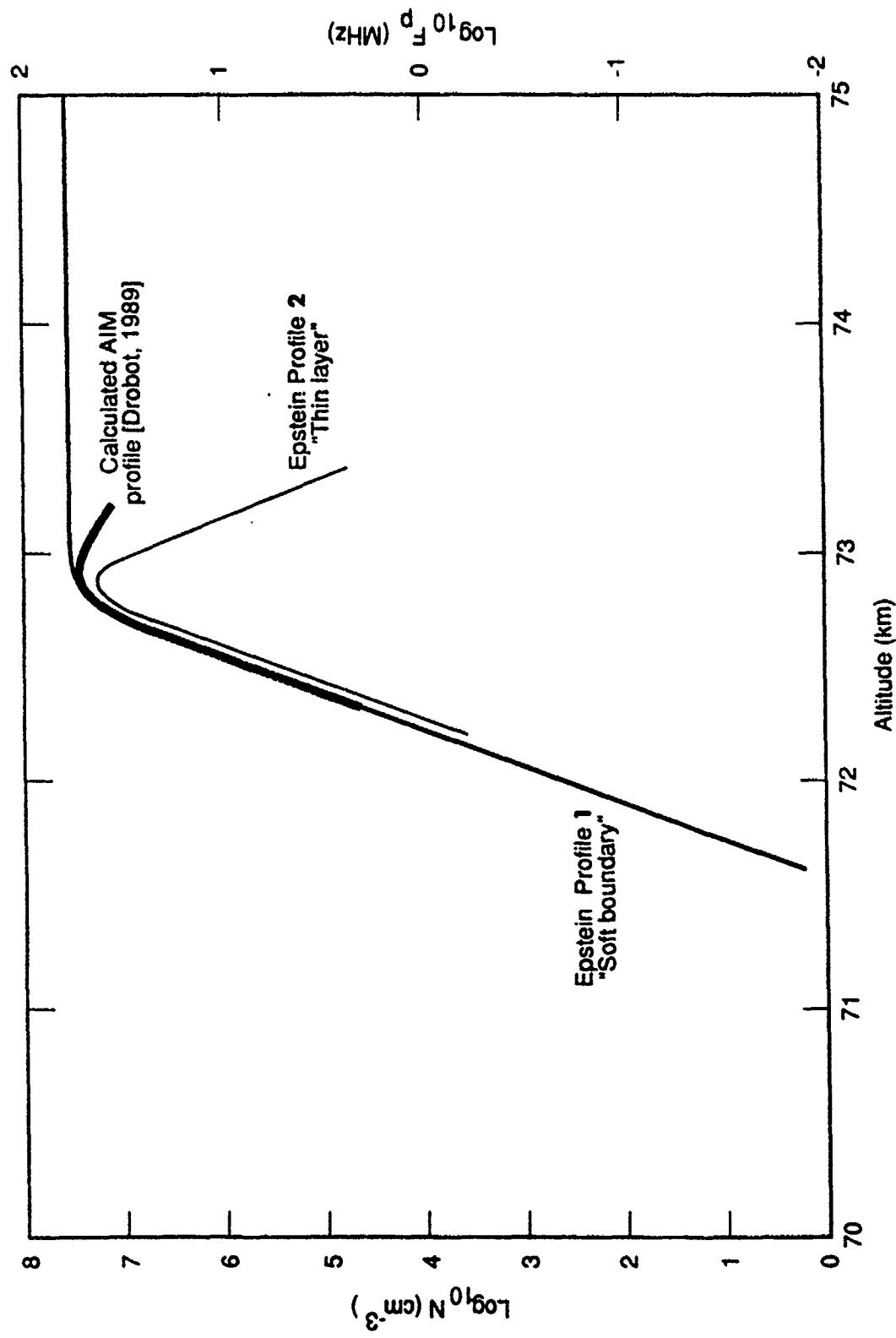


Figure 2. Comparison of Epstein "Thin Layer" and "Soft Boundary" profiles with Calculated AIM Profile peaking at 72.5 km; Boundary Scale $L = 68.6$ m.

This layer differs from the thin one shown in Eqs. (4) and (5) because it is uniform when $z > z_0$. For this reason, we call the layer given by Eq. (8) the Epstein layer with a soft boundary. Examples of that layer are plotted on Figs. 1 and 2. Note that if the scale length L approaches zero, then the electron density jumps abruptly from zero to N_∞ at $z = z_0$, and the refractive index in Eq. (7b) acts as a uniform, isotropic, sharply-bounded half-space.

COMPARISON WITH AIM PROFILES

Drobot [1989] has calculated electron density profiles produced by intense microwave beams having frequencies around 500 MHz. Two of these profiles are shown in Figs. 1 and 2 to illustrate how well Epstein profiles can model actual ones. We see that the Epstein profiles fit the AIM profiles almost perfectly in the lower boundary, but deviate above $z = z_0$ where the thin layer model understates the ionization and the soft boundary model overstates it. We will find, however, that for most cases of practical interest the reflectivities of the thin and soft boundary models are nearly identical. This result shows that reflections are occurring below $z \approx z_0$ where the models accurately represent the real layer. Deviations above $z > z_0$ are not of much concern because in the majority of cases the waves are reflected below that level.

Also note that the AIM layers extend only over a 2 or 3 km altitude range. Because the collision frequency varies with a scale height of 6 or 8 km, it is permissible to use a constant collision frequency, as we have done in our calculations.

SECTION III
REFLECTIVITY FORMULAS

Budden [1961] gives formulas for the reflection and transmission coefficients for the Epstein thin layer and soft boundary. This section reviews these formulas and derives limiting cases of interest.

CASE I: EPSTEIN THIN LAYER (SECH² LAYER)

We consider a wave of frequency f incident from below at angle θ with respect to the vertical. The wavelength is λ , and the wave number k is $2\pi/\lambda$, where c is the speed of light. The reflection coefficient R_1 and transmission coefficient T_1 are

$$|R_1| = \left| \frac{\Gamma\left(\frac{1}{2} + \frac{4\pi i L \cos \theta}{\lambda} - \alpha\right) \Gamma\left(\frac{1}{2} + \frac{4\pi i L \cos \theta}{\lambda} + \alpha\right)}{\Gamma\left(\frac{1}{2} - \alpha\right) \Gamma\left(\frac{1}{2} + \alpha\right)} \right| \quad (9)$$

$$|T_1| = \left| \left(\frac{4\pi L \cos \theta}{\lambda} \right) \frac{\Gamma\left(\frac{1}{2} + \frac{4\pi i L \cos \theta}{\lambda} - \alpha\right) \Gamma\left(\frac{1}{2} + \frac{4\pi i L \cos \theta}{\lambda} + \alpha\right)}{\left[\Gamma\left(1 + \frac{4\pi i L \cos \theta}{\lambda}\right) \right]^2} \right| \quad (10)$$

where we have defined

$$\alpha = \frac{1}{2} \left[1 - \frac{64\pi^2 L^2 X_0}{\lambda^2 (1-iZ_0)} \right]^{\frac{1}{2}} \quad (11)$$

and Γ is the well-known complex gamma function [Abromowitz and Stegun, 1972]. The interpretation of Eq. (9) and (10) is as follows:

$$|R_1^2| = \text{fraction of incident energy reflected} \quad (12a)$$

$$|T_1^2| = \text{fraction of incident energy that leaks through the layer} \quad (12b)$$

$$P = 1 - |R_1^2| - |T_1^2| = \text{fraction of incident energy lost to joule heating in the layer} \quad (12c)$$

Equations (9) and (10) are exact full-wave results, provided that the wave is horizontally polarized. There are no restrictions on frequency, wavelength, or other parameters. Although these equations are easily programmed, it is possible to derive simple asymptotic formulas that apply to nearly all cases of interest and yield good physical insight.

Stirling's approximation to the Γ functions in Eqs. (9) and (10) can be used if the following conditions are met:

$$\frac{\omega_{po}}{\omega(1 + Z_o^2)^{1/4}} \gg \cos \theta \quad (13a)$$

$$\frac{4\pi L}{\lambda} \frac{\omega_{po}}{\omega(1 + Z_o^2)^{1/4}} > 1 \quad (13b)$$

Condition (13a) requires that the maximum plasma frequency in the layer--adjusted for collisions and obliquity--exceeds the wave frequency. Condition (13b) causes $|z|$ to exceed unity and is the usual short-wave criterion for applying the WKB solution.

According to Stirling's formula

$$\Gamma(z) \sim e^{-z} z^{z-1} \sqrt{2\pi} \left(1 + \frac{1}{12z} + \frac{1}{288z^2} + \dots \right) \quad (14)$$

so the asymptotic conditions (13a) and (13b) need not be too stringently enforced; even if the argument of Γ is on the order of unity, the error according to Eq. (14) is only around 8 percent.

By inserting Eq. (14) into Eqs. (9) and (10) and retaining the leading term, we find the following simple result:

$$|R_1| \approx \exp \left(- \frac{4\pi L}{\lambda} \cos \theta \operatorname{Arctan} \nu/\omega \right) . \quad (15)$$

which is the well-known full-wave result for reflection from a profile having an exponentially increasing electron density [Budden, 1961]. It is also the result obtained by using the phase-integral method. Therefore, when conditions (13a) and (13b) are satisfied, all of the interaction between the wave and the layer occurs below the level $z = z_0$, where the profile is exponential.

Equation (15) can be simplified even further in the limits of strong or weak collisions. If $\nu/\omega \ll 1$, then $\operatorname{Arctan} \nu/\omega \approx \nu/\omega$, and Eq. (15) becomes

$$|R_1| \approx \exp \left(- \frac{4\pi L}{\lambda} \frac{\nu}{\omega} \cos \theta \right) \quad \left. \right\} \text{if } \nu/\omega \ll 1 . \quad (16a)$$

$$\text{or} \quad |R_1| \approx \exp \left(- \frac{2L\nu}{c} \cos \theta \right) \quad \left. \right\} \quad (16b)$$

If, on the other hand, $\nu/\omega \gg 1$, then $\operatorname{Arctan} \nu/\omega \approx \pi/2$, and Eq. becomes

$$|R_1| \approx \exp \left(- \frac{2\pi^2 L \cos \theta}{\lambda} \right) \quad \text{if } \nu/\omega \gg 1 . \quad (17)$$

Equation (17) is the well-known expression for the reflection of horizontally polarized VLF waves from a collision-dominated exponential refractive index [Wait, 1970]. given by

$$n^2 \approx 1 - \frac{i\omega_{po}^2}{\omega\nu_0} \exp \left(\frac{z - z_0}{L} \right) . \quad (18)$$

Equation (18), with $\beta = 1/L$ and $z_0 = H$, is often called the " βH "

approximation to the refractive index and is widely used in long wave propagation calculations.

CASE II: EPSTEIN SOFT BOUNDARY

The reflection coefficient for the soft boundary profile given by Eq. (8) is

$$|R_2| = [R_\infty(\cos \theta)][D(L/\lambda)] \quad (19)$$

where

$$R_\infty = \frac{q_\infty - \cos \theta}{q_\infty + \cos \theta} \quad (20)$$

is the Fresnel coefficient for reflection from a sharply-bounded half-space, where the refractive-index jumps abruptly from unity (free space) to N_∞ (within the half-space). The quantity

$$D = \left| \frac{\Gamma^2 \left[1 + \frac{2\pi i L}{\lambda} (q_\infty + \cos \theta) \right]}{\Gamma^2 \left[1 + \frac{2\pi i L}{\lambda} (q_\infty - \cos \theta) \right]} \right| \quad (21)$$

is a diffusivity factor that reduces the reflectivity because of the gradualness of the boundary. Two limiting cases are useful:

Sharp Boundary: $L/\lambda \ll 1$

$$D \rightarrow 0 ; R_2 \rightarrow R_\infty \quad (22)$$

Soft Boundary:* $(2\pi L/\lambda) \left(\frac{\omega_{po}^2}{\omega^2} \sqrt{1 + \nu^2/\omega^2} \right) > 1$

$$|R_2| \approx e^{-\frac{4\pi L \cos \theta}{\lambda} \operatorname{Arctan} \nu/\omega} \quad (23)$$

*Note that because of the different functional forms for Eqs. (4) and (7), the value ω_{po}^2 for the thin layer is half ω_{po}^2 for the soft boundary profile.

We see that the Fresnel coefficient is recovered for the sharp boundary, and that for the gentle boundary, Eq. (23) for R_2 is identical with Eq. (15) for R_1 . That result is as it should be, because in both cases the reflection occurs below $z = z_0$, where the thin layer and soft boundary Epstein profiles are identical.

In summary, we have derived exact equations for the reflection coefficient of the Epstein thin layer [Eqs. (9) and (10)] and the Epstein soft boundary profile [Eqs. (19) and (21)]. These equations are easily programmed and should be used for general cases. However, subject to the often-satisfied conditions (13a) and (13b), the following simple results apply to either profile:

$$|R| \approx \exp \left(-\frac{4\pi L}{\lambda} \cos \theta \operatorname{Arctan} \nu/\omega \right) \quad (24)$$

$$\left. \begin{aligned} &\approx \exp \left(-\frac{4\pi L}{\lambda} \frac{\nu}{\omega} \cos \theta \right) \\ &= \exp \left(-\frac{2L\nu}{c} \cos \theta \right) \end{aligned} \right\} \nu/\omega \ll 1 \quad (24a)$$

$$\approx \exp \left(-\frac{2\pi^2 L}{\lambda} \cos \theta \right) \quad \nu/\omega \gg 1 \quad . \quad (24b)$$

SECTION IV
A SIMPLE CRITERION FOR GOOD REFLECTION FROM
OVERDENSE LAYERS WITH GRADUAL BOUNDARIES

An artificial layer will almost always be designed so its plasma frequency at least somewhat exceeds the frequency of the wave to be reflected. In that case condition (13a) is satisfied, and Eqs. (24), (24a), and (24b) for the reflection coefficient are valid, provided that

$$\frac{4\pi L}{\lambda} \gtrsim 1 \quad , \quad (25)$$

which will be satisfied for any realistic case in the HF/VHF bands. We can therefore use those simple equations to establish criteria for good reflectivity.

Let us somewhat arbitrarily require that the reflection loss be no greater than 20 dB. In order to meet that requirement, the following conditions must be met:

$$(2)(8.66)\pi^2 \frac{L}{\lambda} \cos \theta \leq 20 \quad \text{if} \quad \nu/\omega \gg 1 \quad (26)$$

$$\left. \begin{array}{l} (4)(8.66)\pi \frac{L}{\lambda} \frac{\nu}{\omega} \cos \theta \leq 20 \\ \text{or} \\ \frac{(2)(8.66)}{3 \times 10^8} L\nu \cos \theta \leq 20 \end{array} \right\} \quad \text{if} \quad \nu/\omega \ll 1 \quad . \quad (27a)$$

$$(27b)$$

By inserting numerical values into Eq. (26), we find

$$\frac{L}{\lambda} \cos \theta \leq 0.12 \quad \text{if} \quad \nu/\omega \gg 1 \quad . \quad (28)$$

Because we are interested in wavelengths, λ , on the order of 10 m, Eq. (28) requires that the layer boundary be narrower than about 1 m. Such an abrupt boundary would be virtually impossible to produce and

maintain, so condition (28) implies that the collision frequency ν must be smaller than the angular frequency ω .

We therefore must use the condition (27a) or (27b), which are equivalent to each other. Insertion of numerical values gives

$$\left\{ \begin{array}{l} \left(\frac{L}{\lambda} \right) \left(\frac{\nu}{\omega} \right) \cos \theta \leq 0.2 \\ \text{or} \end{array} \right. \quad (29a)$$

$$\left. \begin{array}{l} L \nu \cos \theta \leq 3.5 \times 10^8 \\ \text{if } \nu/\omega < 1 \end{array} \right. \quad (29b)$$

Either condition (29a) or (29b) will guarantee reflection loss no greater than 20 dB, subject to our assumption that the layer is stratified.

In order to see what the above conditions imply, note that $L \approx 70$ m for the layers shown in Figs. 1 and 2. In that case the requirement for good reflectivity becomes

$$\nu \cos \theta < 5 \times 10^6 \text{ Hz} . \quad (30)$$

In order to use Eq. (30) we must distinguish between conditions when the intense ionizing beam is on and when the beam is off. When the beam is off, the ambient collision frequency ν should be used. When the beam is on the electrons become energetic, and the collision frequency can exceed its ambient value by a factor of 35 [Borisov, Gurevich, and Milikh, 1988].

Under ambient ("beam-off") conditions, the electron collision frequency is less than 5×10^6 Hz, provided the altitude exceeds about 70 km. The condition for good reflectivity therefore requires that the layer be produced above 70 km.

Under "beam-on" conditions the collision frequency exceeds 5×10^6 Hz, unless the altitude is so high that the ambient collision frequency is less than 1.5×10^4 Hz. This condition requires that the layer be produced at an altitude no lower than 110 km, if the ionizing beam is left on during the reflection process.

It is in fact believed that the layer can be quickly formed and will persist for a half-second or more after the ionizing beam is

turned off. If that belief is true, the layer could be produced at an altitude as low as 70 to 80 km and still offer acceptable reflectivity.

SECTION V NUMERICAL RESULTS

This section plots full-wave reflection coefficients calculated from Eqs. (9) and (10) for the Epstein thin layer, and from Eqs. (19) to (21) for the Epstein soft boundary profile. The parameters used as inputs are shown in Figs. 1 and 2. Most of these results conform closely to the simple approximate formulas given by Eq. (25). However, near the high end of the 5 to 40 MHz band considered, the frequency approaches or exceeds the layer's plasma frequency and violates condition (13a). In such instances, the graphs in this section must be used because the simple approximate formulas break down.

Figure 3 shows reflection coefficients for layers centered at an altitude of 61 km. We assume beam-off conditions. The reflectivity is extremely poor--an expected result since the ambient collision frequency at 61 km is 1.7×10^7 Hz, which exceeds the 5×10^6 Hz limit established in Sec. IV.

Figure 4 shows reflection coefficients for beam-off condition layers centered at an altitude of 72.5 km. The reflection loss is marginally acceptable--also an expected result since the collision frequency at 72.5 km is 3×10^6 Hz, slightly less than the 5×10^6 Hz limit discussed above. The coefficients for the soft boundary and thin layer models are nearly identical for frequencies below 25 MHz, as expected, but become different from one another at a frequency of 40 MHz, which approaches the maximum plasma frequency of 50 MHz.

Figure 5 shows reflection coefficients for beam-off conditions, with the layers shown in Fig. 2 moved upward so they are centered at an altitude of 80 km. This procedure is not strictly correct because layer shape depends on altitude, but it gives an idea of the effect of altitude on reflectivity. The reflectivity is high because the collision frequency of 10^6 Hz is substantially below the 5×10^6 Hz limit.

Figure 6 plots the reflection coefficient and relative joule heating loss P for a thin layer at an altitude of 80 km. The fre-

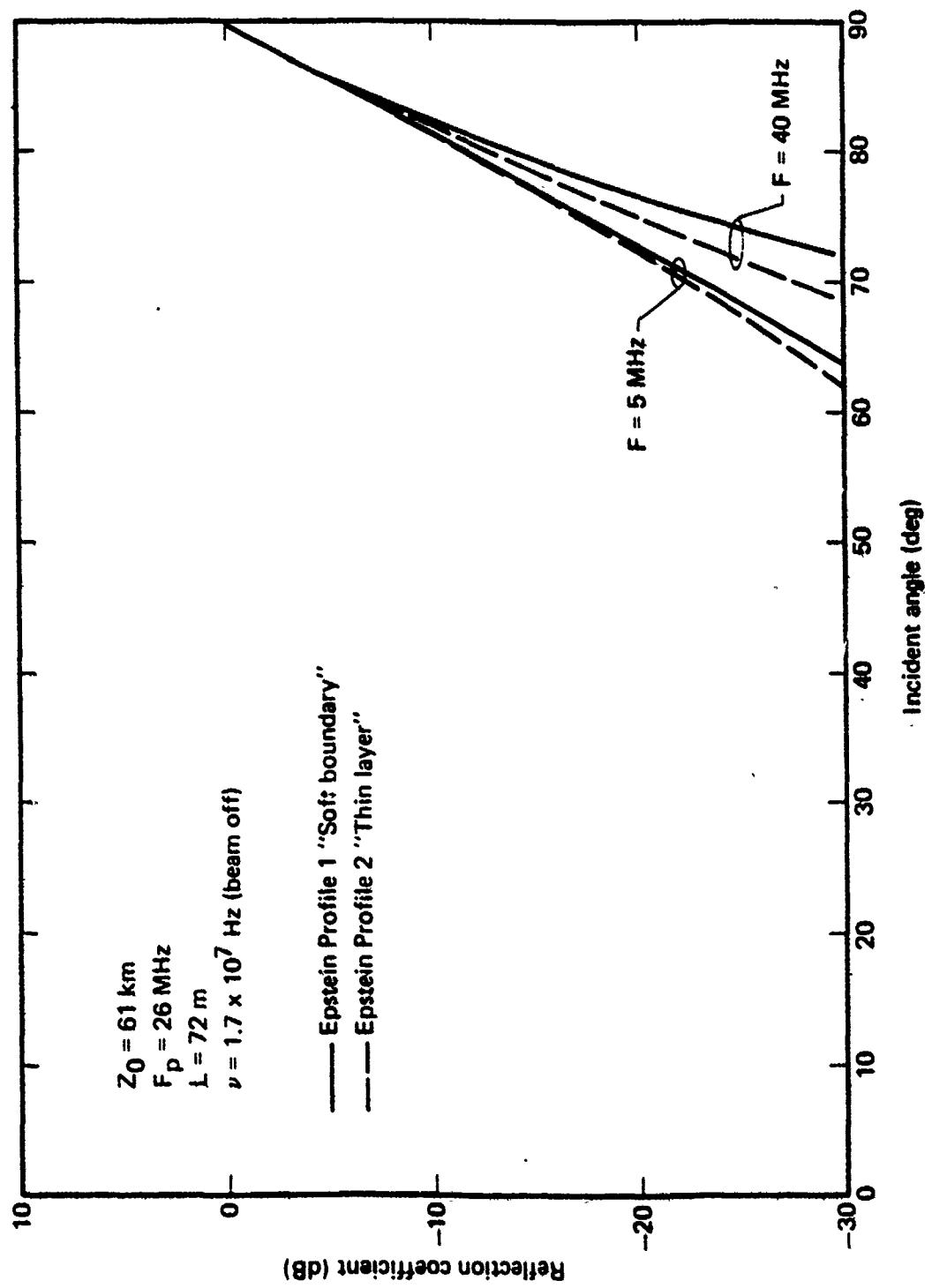


Figure 3. Reflection coefficients for layers at Altitude of 61.4 km.

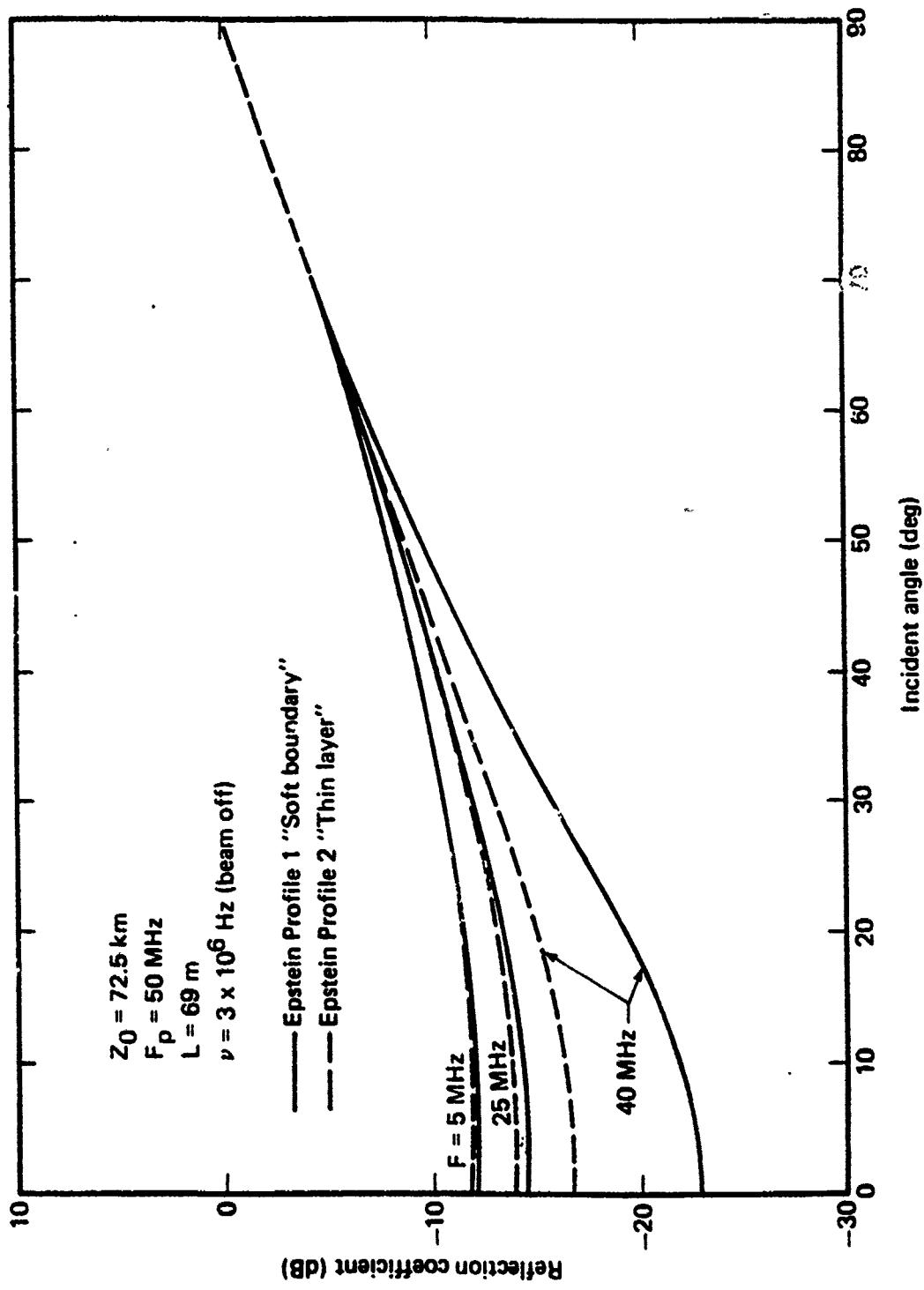


Figure 4. Reflection Coefficients for Layers at Altitude of 72.5 km.

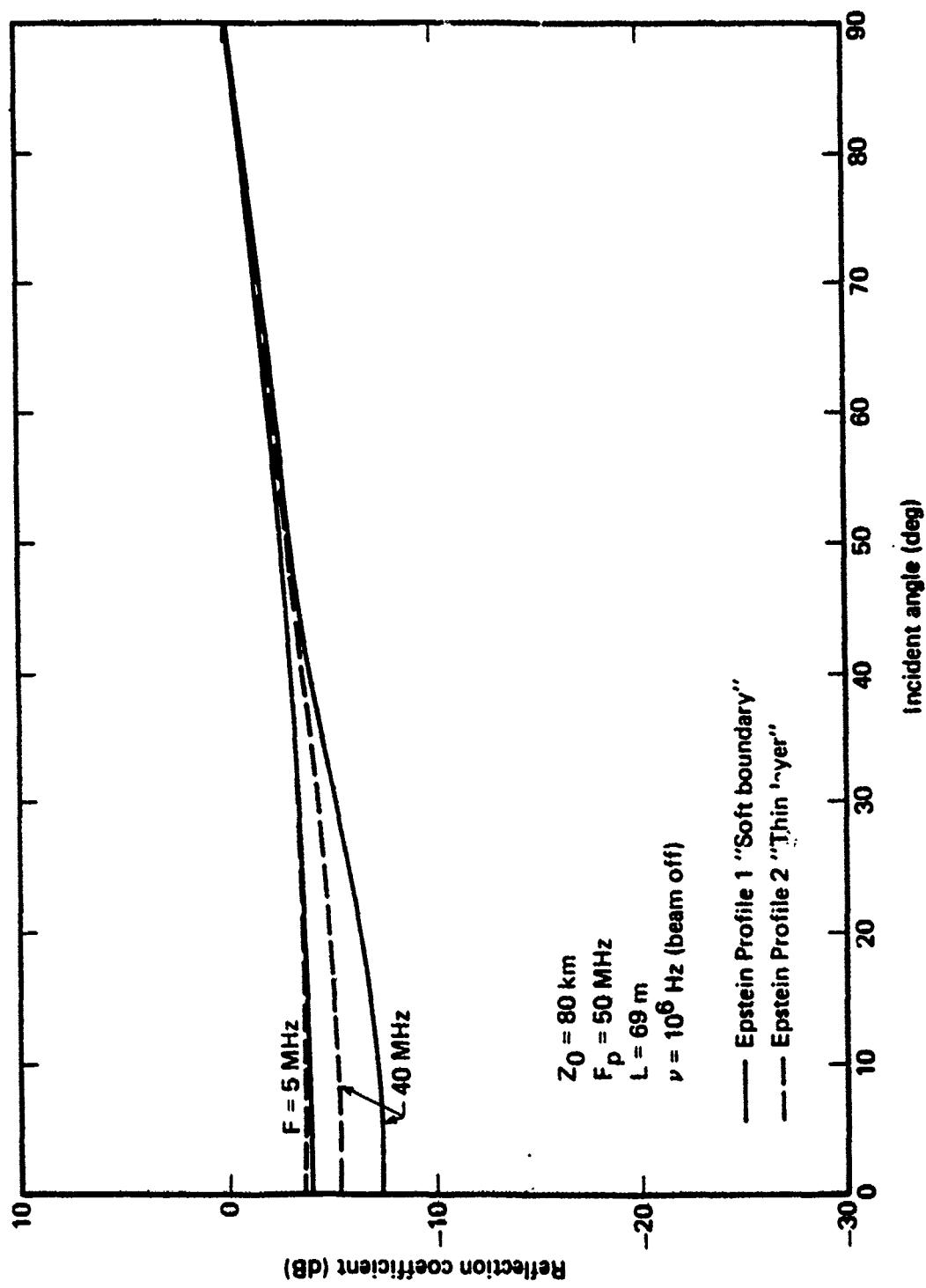


Figure 5. Reflection Coefficients for Layers Shown in Fig. 2 raised to Altitude of 80 km.

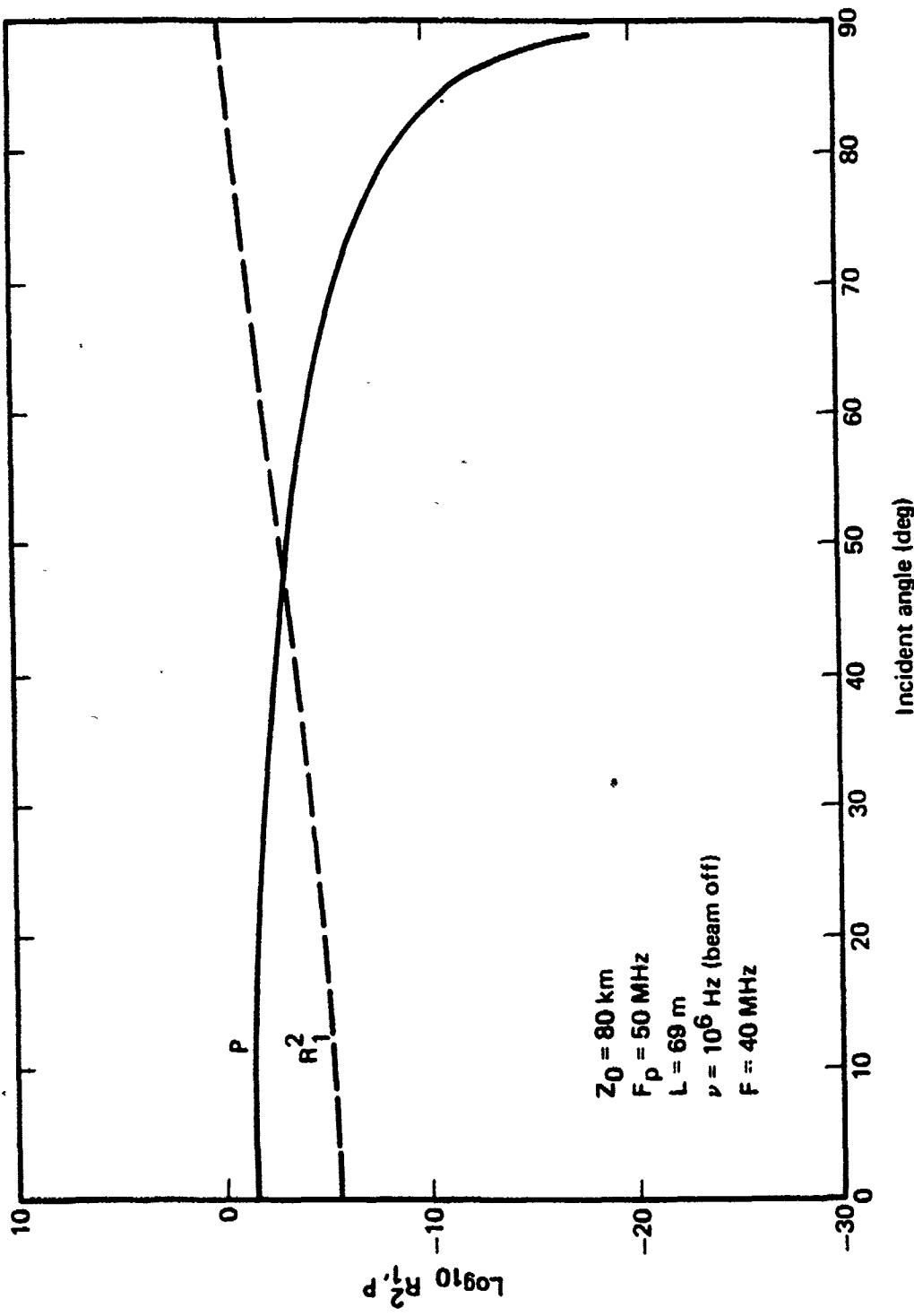


Figure 6. Reflection Coefficient R_1 and Joule Heating Factor P for Epstein Thin Layer at Altitude of 80 km; Frequency = 40 MHz.

quency is 40 MHz and beam-off conditions are assumed. The transmission coefficient T_1^2 is so small that it does not even appear on the plot. It is therefore evident that the reflection loss is due almost entirely to joule heating in the boundary rather than to leakage through the layer. Leakage would be even less important for layers at altitudes below 80 km.

Figures 7 and 8 compare reflection coefficients for beam-off and beam-on conditions for the layers of Fig. 2 centered at altitudes of 72.5 km and 90 km. As the graph illustrates, leaving the beam on has a devastating effect on reflectivity, even at an altitude of 90 km. As shown analytically in Sec. IV, the layer must be at an altitude of at least 110 km if good reflectivity is to be achieved with the ionizing beam on.

We again emphasize that, at frequencies above about 30 MHz, the layers assumed in this section do not have a high enough plasma frequency to be considered truly overdense. In practice, the layer's plasma frequency would probably be chosen to be at least double the frequency of the wave to be reflected. In that case the simple formulas given in Eq. (24) are almost always valid, and the criteria established in Sec. IV for good reflectivity would be universally valid.

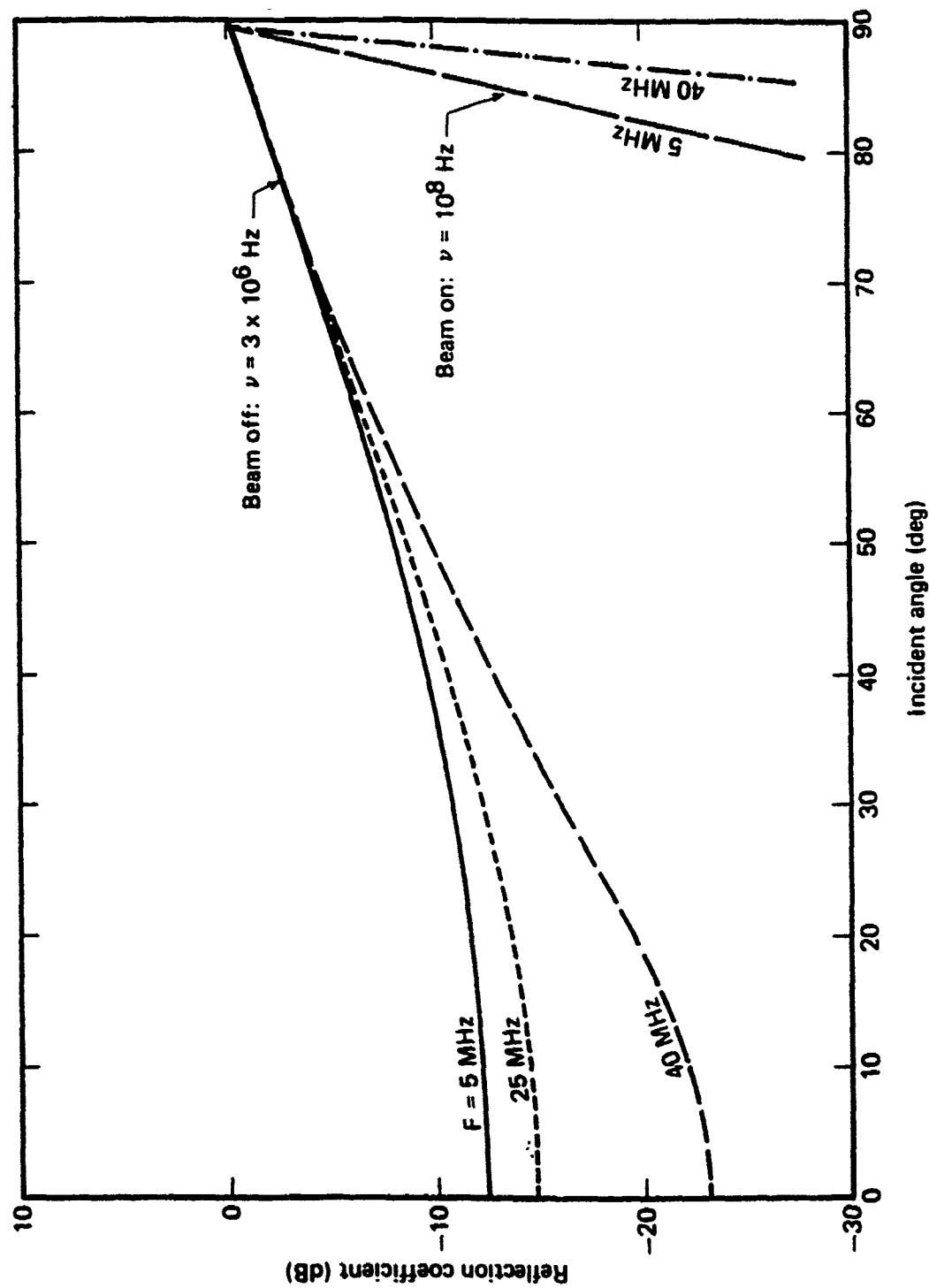


Figure 7. Comparison of Reflection Coefficients for Beam-off and Beam-on Conditions: Epstein Soft Boundary Model at Altitude of 72.5 km.

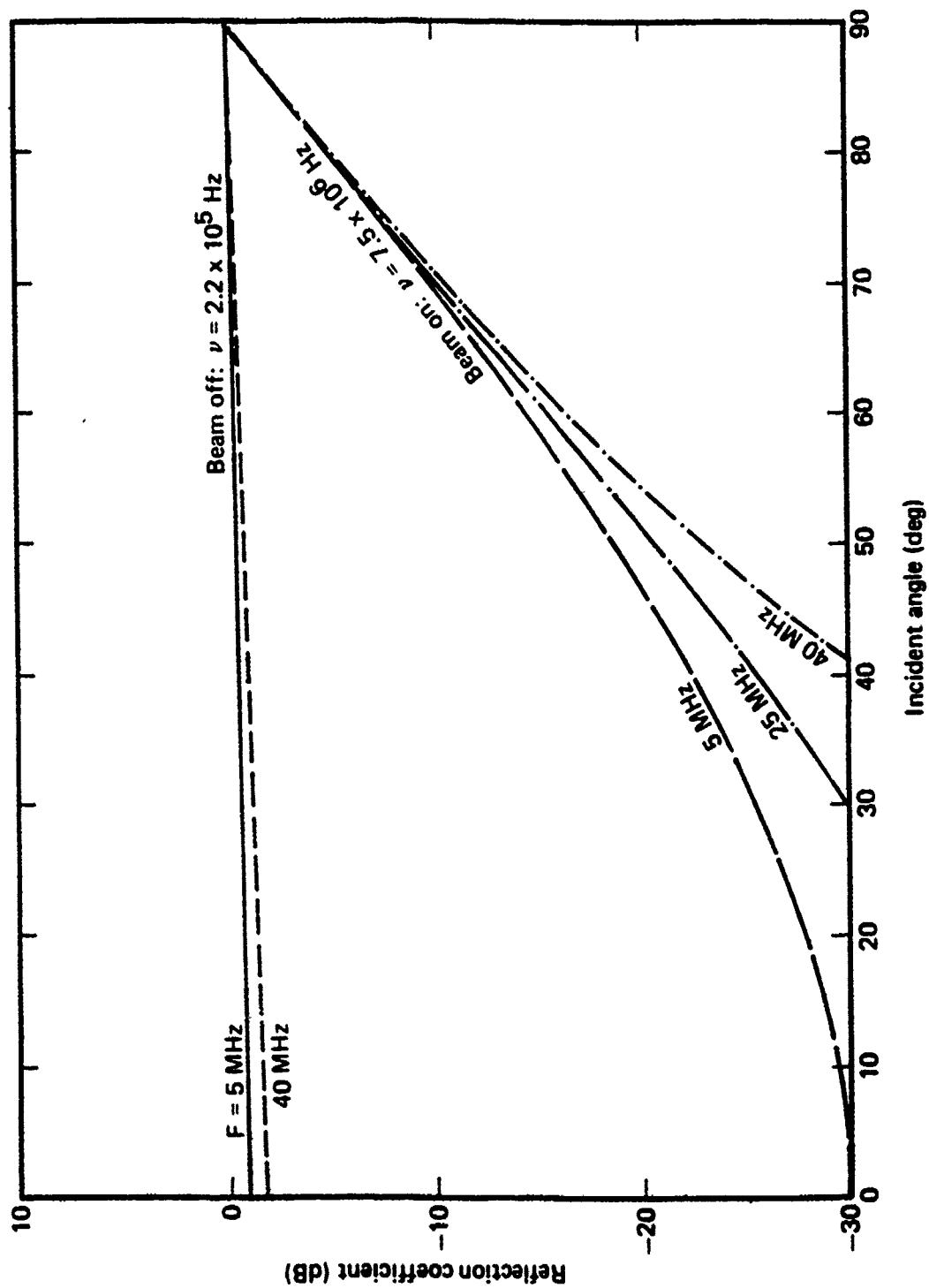


Figure 8. Comparison of Reflection Coefficients for Beam-off and Beam-on Conditions: Epstein Soft Boundary Model, raised from 72.5 km Altitude to 90 km.

SECTION VI
CONCLUSIONS

For overdense plasma layers having plasma frequencies substantially greater than the frequency of the wave to be reflected, the reflection loss is approximately

$$\text{Reflection loss} \approx -5.8 \times 10^{-8} \nu L \cos \theta \quad \text{db}$$

where ν is the electron collision frequency at the layer altitude. $1/L$ is the height-gradient of the electron density in the layer boundary, and θ is the angle of incidence. This condition is nearly universal and depends only on the layer being stratified, the wave being horizontally polarized, and $\nu/2\pi f$ being no greater than unity, where f is the wave frequency.

For layers created with intense 500 MHz beams, the scale length L equals approximately 70 m, and reflection losses greater than 20 dB will be suffered unless the layer altitude is great enough so that the collision frequency ν does not exceed about 5×10^6 Hz. If the reflection occurs during the time between ionizing pulses, collision frequency would require that the layer be at or above about 70 to 75 km. If, however, reflection must occur during the pulse--i.e., when the ionizing beam is on--then the layer must be at an altitude of at least 110 km to avoid prohibitive reflection loss.

Our calculations assume stratified layers and therefore neglect roughness, which could be caused, for example, by uneven illumination of the layer by the ionizing beam. Because such roughness would probably degrade the reflectivity, our conditions for good reflection should be regarded as necessary--but not necessarily sufficient.

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